Dual-Tone Radio Interferometric Positioning Systems for Multi-Target Localization Using a Single Mobile Anchor

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Abstract: In this paper, a low-cost dual-tone radio interferometric positioning system using a single mobile anchor is proposed to locate multiple targets at the same time. We name it mDRIPS. In mDRIPS, each target continuously transmits a dual-tone signal with its dedicated frequencies, and the mobile anchor receives the signals at different positions along its trajectory. Neither time synchronization between the mobile anchor and the targets nor time synchronization among the targets is required. We take the instability of targets clocks into consideration and develop an ESPRIT-type algorithm to estimate frequencies of each dual-tone signal. Furthermore, the time of arrival (TOA) of each target signal is extracted from the phase estimates of the received dual-tone signals. After measuring several TOAs at different locations along the anchor’s trajectory, each target can be located. Since the frequency difference of the two tones of each dual-tone signal is designed to be smaller than the channel coherence bandwidth, the same fading effect on these two tones can be eliminated. Moreover, the integer ambiguity problem due to phase wrapping is investigated, and a localization algorithm to deal with a simplified ambiguity problem is proposed. Numerical results demonstrate the efficiency of the proposed mDRIPS.

Keywords: ranging; localization; mobile anchor; radio interferometry; oscillator instability

I. INTRODUCTION

Location awareness has received lots of interest in many wireless systems such as cellular networks [1], wireless local area networks [2], and wireless sensor networks [3]. The positions of wireless terminals are indispensable for location-based services. For example, search and rescue operations cannot be fulfilled without accurate navigation. Moreover, a large set of wireless network applications [4] require device locations to meaningfully interpret the collected data. The high cost of ultra-wideband (UWB) localization systems [5] discourage their popularity even with high localization accuracy. Therefore, accurate and low-cost localization techniques are crucial for the location-based applications with cost-constrained terminals.

Recently, radio interferometric techniques are used for localization by measuring the relative phase offset of interfering radio signals. Maróti et al. propose a radio interferometric
positioning system (RIPS) [6] to achieve both high accuracy and low cost. The RIPS uses a radio signal strength indicator (RSSI) to measure the low-frequency differential signal, which is generated by two interfering radios, and estimates its phase to extract range information. However, the RIPS only accommodates additive white Gaussian noise (AWGN) channels, retains approximation errors, and faces the integer ambiguity issue. The RIPS is further extended in [7], [8] to track mobile nodes, where Doppler shifts are explored and velocity estimates of moving targets are achieved. Spinning anchors transmitting radio signals with fixed frequencies (SpinLoc) are employed in [9] to produce specified Doppler signals, and then angle of arrivals (AOAs) are estimated as localization metrics. An asynchronous RIPS (ARIPS) is proposed in [10], where time-difference-of-arrival (TDOA) based on radio interferometric technique is employed to localize asynchronous targets. Nevertheless, the ARIPS is still for AWGN channels and maintains the similar approximation errors as the original RIPS. Recently, a dual-tone radio interferometric positioning system using undersampling techniques (uDRIPS) is proposed in [11], and it is immune to flat fading channels. By using the directly undersampling techniques, uDRIPS can avoid the amplification of measurement noise and simplify the receiver structure. However, uDRIPS does not consider the instability of target clock. Moreover, the localization techniques of the above papers [6]-[11] all require several anchors to be synchronized and cooperate with each other, which increases the cost and communication overheads. For example, the RIPS relies on interferometric measurements using a large number of combinations of four nodes. These measurements are carried out in a sequential manner. Further, the data needs to be routed back to a fusion center to apply the localization algorithm. The entire process is time-consuming.

To reduce overheads due to multi-anchor localization, the relevant work in [12]-[15] also considers single-anchor localization techniques. A single-anchor indoor localization method is proposed in [12] using UWB signals to distinguish different signal paths to locate a mobile target. Other single-anchor indoor localization systems proposed in [13], [14] use smart antennas to estimate the AOAs of the target. Nevertheless, UWB techniques and smart antenna strategies suffer from high cost. Moreover, a single receiver emitter geolocation system taking the instability of clock into consideration is developed in [15] based on periodic signals. However, AWGN channels are implicitly considered.

In [16], we propose a low-cost dual-tone radio interferometric positioning system using a single mobile anchor (mDRIPS) in the presence of noise, flat-fading channels and the target oscillator instability. In this paper, we develop mDRIPS from a single target localization application into a multiple targets localization system. No communication and synchronization overheads are generated, since only a single anchor is used for locating the targets. Hence, manipulating a single receiver is cost-efficient than operating several receivers simultaneously. In mDRIPS, each target continuously transmits a dual-tone signal without synchronization, and the influence of the target clock offset can be eliminated. However, the target clock skew caused by oscillator instability has an effect on the phase estimation of the received dual-tone signal. The clock skew of each target will lead to an extra frequency offset of the received two tones, and thus the phase estimation based on the assigned frequencies is not accurate. To solve the clock skew problem caused by oscillator instability, an ESPRIT-type algorithm is designed to estimate the tone frequencies of all targets, and then the clock skew can be calculated based on the estimated tone frequencies. Further, the time of arrival (TOA) of each target can be extracted from the phase of the received dual-tone signal using the estimated frequencies. After measuring several TOAs at different positions along the anchor’s trajectory, each target’s location can be estimated by the mobile anchor. Since the frequencies
in Fig. 1, the mobile anchor moves in a periodic manner with a time-slot of duration $T$. Each time slot is divided into receiving and moving mini slots. Each target continuously transmits a dual-tone signal, and we assume $I$ targets to be located. The mobile anchor moves to a new position by every slot $T$, and receives each target signal at the receiving mini slot. The duration of the receiving mini-slot is $T_p$. The received signals at location $s_i$ will be used to estimate the received tone frequencies of each target. They are used for the subsequent TOA estimation. The receiving and moving steps repeat until several TOA measurements for each target are collected. Finally, the localization algorithm is adopted to locate each target based on the estimated TOAs.

The anchor clock is assumed to be calibrated and thus accurate. Taking the anchor clock as a reference, similar to [17], [18], the clock model of the $i$th target is

$$c_i(t) = w_i t + \Delta_i, \quad (1)$$

where $w_i \triangleq 1 + \mu_i$, and is the $i$th target clock skew with respect to (w.r.t.) the anchor clock, $\Delta_i$ is the initial clock offset, and $c_i(t)$ is the $i$th target local time relative to the anchor’s clock. Note that $\mu_i$ is typically very small, measured in parts per million (ppm). It is well known that the obvious time-varying clock skew phenomenon could be observed over day timescale for highly reliable CPU oscillators and seconds or minutes timescales for low-cost wireless sensors [18]. The localization latency of mDRIPS is about several minutes. Hence, we assume that the target clock skew $w_i$ is invariant during each localization procedure.

The dual-tone signal transmitted by the $i$th target is

$$s_i(t) = a_i e^{j 2 \pi f_b (t - T_p)} (1 + e^{j 2 \pi f_s t}), \quad (2)$$

where $a_i$ is the real-valued amplitude of the dual-tone signal, $f_b$ is the carrier frequency, $f_s$ is the basic frequency assigned to the $i$th target and greater than zero, and $g_s$ is the small frequency difference between the two tones. Since $g_s$ is designed to be smaller than the channel coherence bandwidth, the two tones of each target will experience the same chan-
nel fading effect [19]. As a result, a flat-fading channel model is applied to account for the fading effect. We assign different basic frequencies to different targets to help the receiver distinguish them easily, and $g_s$ is the same for all targets.

In mDRIPS, a mobile anchor is adopted to receive the signals of multiple targets, estimates the corresponding TOAs of each target at $N$ different positions $s_i, s_1, ..., s_{N-1}$ along its trajectory. At the first position $s_i$, the time instant of mobile anchor beginning to receive the signals is $t_0$. We assume that all targets start to transmit their dual-tone signals before $t_0$ and $t_0$ can be determined by the mobile anchor. When the anchor arrives at the $l$th position, this instant can be calculated as

$$t_l = t_0 + lT, \quad 0 \leq l \leq N - 1.$$  

(3)

Without loss of generality, the received signals of the anchor at the $l$th position are down converted by $f_l$, and can be modeled as

$$r_l(t) = \sum_{i=1}^{L} \beta_{kl} (t - t_l) e^{-j2\pi f_i t} \quad t_1 \leq t \leq T_2$$

(4)

where $\beta_{kl}$ is the complex channel coefficient for the flat-fading channel, and can be modeled as a zero-mean complex Gaussian random variable with variance $\sigma_{kl}^2$, representing the average power of the flat-fading channel. Moreover, $t_l$ is the time instant of the mobile anchor when it arrives at the $l$th position, and $t_0$ is the unknown propagation delay. It is related to the distance as $d_{ll} = ct_{ll}$, where $c$ is the signal propagation speed, and $d_{ll}$ is the distance between the $l$th target and the $l$th position of the mobile anchor. Note that for simplicity noise is neglected in (4). However, the proposed algorithm is able to accommodate any noise.

Plugging (2) to (4), we obtain

$$r_l(t) = \sum_{i=1}^{L} \alpha_{kl} (e^{j2\pi f_i t - \theta_{kl}} + e^{j2\pi f_i (t - T_i)}).$$

(5)

where $t_1 \leq t \leq t_l$, $f_i' = f_i + f_{i0}$, $f_i'' = f_i - f_{i0}$, $\omega_{i} = 2\pi f_i (t_l - T_i)$, $\alpha_{i} = a_i e^{jw_{i0}(t_l - T_i)}$. Observe that the received tone frequencies of the $i$th target $f_i', f_i''$ are different from the assigned frequencies $f_i + g_{i}$ because of the clock skew. The accurate phase information $\varphi_i$ cannot be obtained without the exact estimation of $f_i', f_i''$. Hence, in Section III, the clock skew problem will be solved and the phase information will be estimated sequentially.

**III. RANGE ESTIMATION IN THE PRESENCE OF OSCILLATOR INSTABILITY**

The received dual-tone signals $r_l(t)$ is directly sampled with Nyquist rate $f_s$ and $M$ samples are collected into a vector $r_l$ as follows

$$r_l = H_l y_l,$$

(6)

where $y_l = [a_0, b_0, a_1, b_1, ..., a_{M-1}, b_{M-1}]^T$, $a_i = a_i e^{j\theta_i}, b_i = a_i e^{j\theta_i}$, $H_l = \{\Phi(f_1', f_l, t_l), \Phi(f_1'', f_l, t_l), ..., \Phi(f_{M-1}', f_l, t_l), \Phi(f_{M-1}'', f_l, t_l)\}$.

According to (6), there are $2M$ different tones included in $r_l(t)$. To avoid aliasing of the different tones, we first develop the frequency allocation method for all targets.

**3.1 Frequency allocation**

As shown in Fig. 2 (a), each target will be assigned a dedicated dual-tone signal with two tones. The frequencies of the two tones are $f_i'$ and $f_i'' + g_{i}$. The dual-tone signal will experience a flat-fading channel since we design $g_{i}$ to be smaller than the channel coherence bandwidth. Without the clock skew, the received tones of the mobile anchor will be the same as the nominated or assigned frequencies. However, as shown in Fig. 2 (b), the received frequencies for each target is different from the nominated frequencies and there is a frequency offset between them. According to (5), $f_i'$ and $f_i''$ are two tones for the $i$th target. To avoid aliasing of different dual-tone signals, we should keep $f_i'' > f_i'$. This can be easily ensured by allocating $f_i'' - f_i > g_{i} + 2\Delta_{i}$, where $g_{i}$ is the maximum frequency offset produced by clock instability.

The tone frequency $f_i''$ is the largest one.
in mDRIPS. We remark that here the Nyquist rate ($f_s > 2f_{c,n}$) is small according to the design of the tone frequencies, which is easy for handling even with cost-constraint devices. For example, when $f_{c,n} \approx 1$ MHz, $f_s \approx 434$ MHz, $g_n = 50$ KHz, $\mu = 10$ ppm, we can obtain $f_{c,n} = 1.054$ MHz, and $f_s$ can be set 3 MHz. Moreover, instead of the squaring operation to generate a low-frequency differential signal as [6]-[9], the direct sampling technique adopted in mDRIPS can avoid the auto-correlation term between the noise and the cross-correlation term between the signal and the noise.

Note that in (6) the propagation delay ($r_{i,j}$) is borne by the phase of the received dual-tone signals. The estimates of $\theta_{i,j}$ and $\varphi_{i,j}$ depend on accurate knowledge of the received dual-tone frequencies $f_i^1, f_i^2$. However, we cannot have accurate knowledge of $f_i^1$ and $f_i^2$ due to the unknown clock skew of the $i$th target. Therefore, we need to estimate frequencies at first, and then the estimated parameters will be used sequentially.

3.2 Frequency estimation: ESPRIT

The ESPRIT algorithm is proposed in [20] to estimate direction-of-arrival (DOA) via rotational invariance techniques. In mDRIPS, an ESPRIT-type algorithm is designed to estimate $f_i^1$ and $f_i^2$ ($i = 0, 1, \ldots, L-1$) based on the received dual-tone signals at the first position ($x_0$) of the mobile anchor. Since the target clock skew is assumed to be invariant during the whole localization session, the estimated frequencies can be used in the rest ranging and localization steps.

As in [10], the sample vector $r_0$ is rearranged into a matrix $R$ of size $p \times q$ with $q = M - p + 1$ as

$$R = \begin{bmatrix} r_{1,1} & r_{1,2} & \cdots & r_{1,M} \\ r_{2,1} & r_{2,2} & \cdots & r_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ r_{p,1} & r_{p,2} & \cdots & r_{p,M} \end{bmatrix},$$

(7)

where $[a]_m^n$ represents the vector composed of the $m$th to $n$th elements of the vector $a$. The matrix $R$ can be factorized as

$$R = \begin{bmatrix} \Sigma \text{diag}(y_0) & \Sigma \text{diag}(y_1) & \cdots & \Sigma \text{diag}(y_{p-1}) \end{bmatrix},$$

(8)

where $[H_0]_p$ represents the first $p$ rows of matrix $H_0$, and the operator $T$ is transposition. When $[H_0]_p$ and $[H_1]_q$ are tall matrices, the rank of $R$ is no greater than $2I$ and $R$ retains the shift-invariant property [20]. We take submatrices of $R$ as

$$R_i = \begin{bmatrix} I_{p-1} & 0_{p-1,1} \end{bmatrix} R = [H_0]_p \text{diag}(y_0) [H_1]_q,$$

$$R_j = \begin{bmatrix} 0_{q-1} & I_{q-1} \end{bmatrix} R = [H_0]_p \Sigma \text{diag}(y_0) [H_1]_q,$$

(9)

where $I_{p-1}$ is the identity matrix with size $p-1$, $0_{p-1,1}$ is a zero vector with length $p-1$ and $\Sigma = \text{diag}(\{e^{j2\pi f_1^1/\omega}, \ldots, e^{j2\pi f_q^1/\omega}\})$.

Therefore, by applying the ESPRIT algorithm based on the special relationship between $R_i$ and $R_j$, we achieve at

$$(R_i^\top R_j)^\dagger = T^\top \Sigma T$$

(10)

where $T = \text{diag}(y_0) [H_1]_q$, and the operator $\dagger$ is pseudo-inverse. The size of $(R_i^\top R_j)$ is $q \times q$, and its rank is $2I$. We apply eigenvalue decomposition and represent the eigenvalue estimates of $(R_i^\top R_j)$ as $\tilde{\lambda}_0, \tilde{\lambda}_1, \ldots, \tilde{\lambda}_{q-1}$ in an ascending order by the absolute values of the eigenvalues. Thus, the frequencies of the $i$th target dual-tone signal can be estimated based on the eigenvalues as follows:

$$f_i^j = \frac{f_s}{2\pi} \arg(\tilde{\lambda}_{2i-1}),$$

(11)
\[
\hat{f}_i = \frac{f_i}{2\pi} \arg \left( \frac{f_i + f_j}{f_i - f_j} \right).
\]

where \( \arg(x) \) is the phase of variable \( x \), and we obtain
\[
\hat{\varphi}_i = \frac{\hat{f}_i}{\hat{f}_j} f_j.
\]

Moreover, \( \mathbf{R}_1 \) and \( \mathbf{R}_2 \) should be low-rank to employ the ESPRIT algorithm. Hence, \( \mathbf{H}_i \) and \( \mathbf{H}_j \) should be strictly tall, \( p > 2l - 1 \) and \( q > 2l \) should be satisfied.

### 3.3 Ranging estimation

With the estimates of \( f_i \) and \( f_j \) for the \( i \)-th target \( (i=0,1,\ldots,l-1) \), we construct
\[
\hat{\mathbf{H}}_i = \left[ \Phi(f_i, f_j, t_i), \ldots, \Phi(f_i, f_j, t_l) \right].
\]

Note that \( r_i \) is the received signal vector of the mobile anchor’s \( i \)-th measurement and the estimation of \( y_i \) can be obtained through a least-squares (LS) estimator
\[
\hat{y}_i = (\hat{\mathbf{H}}_i)'r_i.
\]

Based on (13), the phase of interest \( \varphi_i \) can be estimated as
\[
\hat{\varphi}_i = \arg([\hat{y}_i]_{\text{LS}} + [\hat{y}_j]_{\text{LS}}) + 2\pi z,
\]

where the operator \( \ast \) is the complex conjugate, and an unknown integer \( z \) is introduced due to the phase wrapping. We remark here that the effects of initial phases \( \varphi_0 \) and the flat-fading channel \( \hat{\beta}_i \) are eliminated via (14). Hence, the mDRIPS is immune to the uncertainty of initial phases and the flat-fading channel. However, based on the phase estimation \( \hat{\varphi}_i \), we can obtain the biased TOA information, which is coupled with clock offset \( \Delta_\varphi \) and faces the integer ambiguity issue.

### 3.4 Integer ambiguity issue

Since the integer ambiguity problem due to phase wrapping \( [21] \) exists in mDRIPS, the estimate of \( \varphi_0 \) cannot be calculated from (14) because of the unknown integer \( z \). Hence, we can only achieve \( \hat{\varphi}_i \) instead of \( \hat{\varphi}_i \), where \( \hat{\varphi}_i \) is the estimate of \( \varphi_i \) and \( \varphi_i = \varphi_i - 2\pi \left[ \frac{\varphi_i}{2\pi} \right] \). Recall that \( \tau_i \) is coupled with \( \Delta_\tau \), as \( \varphi_i = 2\pi g_i (w, \tau_i - \Delta_\tau). \)

Let us define \( \varepsilon_i = 2\pi g_i (w, \tau_i) \) and \( \delta_i = \varepsilon_i - 2\pi \left[ \frac{\varepsilon_i}{2\pi} \right] \), then we achieve at
\[
\left\{ \begin{array}{ll}
\frac{\delta_i}{2\pi} + \frac{\varepsilon_i}{2\pi}, & 0 \leq \delta_i + \varepsilon_i < 2\pi \\
\frac{\delta_i}{2\pi} + \frac{\varepsilon_i}{2\pi} + 1, & 2\pi \leq \delta_i + \varepsilon_i < 4\pi
\end{array} \right.
\]

(15)

Based on (15) and \( \hat{\varphi}_i = \varphi_i - 2\pi \left[ \frac{\varphi_i}{2\pi} \right] \), we obtain
\[
\hat{\varphi}_i = \left\{ \begin{array}{ll}
\delta_i + \varepsilon_i, & 0 \leq \delta_i + \varepsilon_i < 2\pi \\
\delta_i + \varepsilon_i - 2\pi, & 2\pi \leq \delta_i + \varepsilon_i < 4\pi
\end{array} \right.
\]

(16)

From (16), the phase offset estimation exists two possible values, and we could not confirm the exact value only based on the estimation of \( \hat{\varphi}_i \). In Section IV, the integer ambiguity problem can be solved through simplification and multiple measurements at different positions.

### IV. Localization Algorithm in mDRIPS

For the \( i \)-th target, we observe that \( \varepsilon_i = \varepsilon_i - 2\pi \left[ \frac{\varepsilon_i}{2\pi} \right] = \varepsilon_i - 2\pi k \), where \( k \) is an integer and related to \( \varepsilon_i \). Note that \( \varepsilon_i = 2\pi g_i (w, \tau_i) \). It is the phase offset due to the propagation delay and \( \varepsilon_i = 2\pi g_i (w, \tau_i) / c \).

Given the maximum ranging distance between the target and the mobile anchor, the possible value of \( k \) can be determined. For example, if \( 0 < \varepsilon_i < 2\pi \) equal to \( d_i < c / g_i \), thus \( k = 0 \). Usually \( w_i = 1 \) and \( g_i \) is designed to be small to benefit the dual-tone signal experiencing a flat-fading channel. Given \( g_i = 50 \text{ kHz} \), the maximum ranging distance without ambiguity is 6 km. Moreover, if \( 0 < \varepsilon_i < 4\pi \), the possible value of \( k \) is 0 or 1 and the maximum ranging distance is 12 km given \( g_i = 50 \text{ kHz} \). In mDRIPS, the maximum value of \( k \) is bounded by \( k \) according to the ranging distance requirement.

Hence, (16) can be transformed to
\[ \hat{\varphi}_{i,c} = \begin{cases} h_i + d_{i} - \frac{k \varphi_{i,c}}{g(w_w)}, & 0 \leq \hat{\varphi}_{i,c} < 2\pi \\ h_i + d_{i} - \frac{(k + 1) \varphi_{i,c}}{g(w_w)}, & 2\pi \leq \hat{\varphi}_{i,c} < 4\pi \end{cases} \]

(17)

where \( h_i = \hat{\varphi}_{i,c}/2\pi g(w_w), 0 \leq k \leq k \).

We collect all the phase estimates of the \( i \)th target \( \hat{\varphi}_{i,c} \) into a vector as \( \mathbf{v}_i = c/(2\pi g(w_w) \hat{\varphi}_{i,c}) (\hat{\varphi}_{i,1}, \hat{\varphi}_{i,2}, \ldots, \hat{\varphi}_{i,n_{\lambda}})' \). Consequently, the model of \( \mathbf{v}_i \) can be given by

\[ \mathbf{v}_i = \mathbf{d}_i + b_i \mathbf{1}_n + \frac{c}{g(w_w)} \mathbf{u}_i, \]

(18)

where \( \mathbf{d}_i = (d_{i,1}, \ldots, d_{i,n_{\lambda}})' \) with \( d_{i,j} = ||s_j - x_i|| \), \( s_j \) denotes the coordinate of the \( j \)th position of the mobile anchor, \( x_i \) is the coordinate of the \( i \)th target and \( \mathbf{u}_i \in \Omega^2(\Omega = \{0, -1, \ldots, -(k + 1)\}) \).

It is difficult to solve (18) directly because of the nonlinear relationship with unknown \( x_i \). Hence, we transfer (18) to a linear model at first. Let us assume the number of time slots for locating the targets is small (e.g. \( N=16 \) is enough to locate the targets with high accuracy), and \( k \leq 2 \) for a large ranging area (12 km in mDRIPS). The value of \( u_i \) is in a limited set, and can be enumerated without high complexity. Hence, \( u_i \) is categorized as a known subset, while the unknown parameters \( x_i \) and \( b_i \) as the other. Moving \( c/g(w_w) \mathbf{u}_i \) and \( b_i \mathbf{1}_n \) to the left side of (18) and making element-wise multiplication, we obtain a linear model with the unknown parameters as

\[ \mathbf{\Theta} - \hat{\mathbf{v}}_i \odot \hat{\mathbf{v}}_i = 2S' \mathbf{x}_i - 2b_i \hat{\mathbf{v}}_i + (b_i^2 - ||s_j||^2) \mathbf{1}_n, \]

(19)

where \( \hat{\mathbf{v}}_i = \mathbf{v}_i - c/g(w_w) \mathbf{u}_i \), \( S = [s_1, s_2, \ldots, s_{n_{\lambda}}] \), and \( \mathbf{\Theta} = [||s_1||^2, ||s_2||^2, \ldots, ||s_{n_{\lambda}}||^2]' \). We apply a brute-force search for all possible values of \( u_i \). The \( n_i \) candidate of \( u_i \) is denoted by \( u_i^{(n_i)} \). With the candidate \( u_i^{(n_i)} \), the corresponding estimates of \( x_i^{(n_i)} \) and \( b_i^{(n_i)} \) can be achieved based on (19) using an LS estimator. Substituting \( x_i^{(n_i)} \) and \( b_i^{(n_i)} \) into (18), the estimation of \( \hat{\mathbf{u}}_i^{(n_i)} \) can be calculated by a simple rounding operation as

\[ \hat{\mathbf{u}}_i^{(n_i)} = \begin{cases} \mathbf{0}, & \text{if } h \geq 0 \\ -(x + 1), & \text{if } h < -(x + 1) \end{cases} \]

(20)

where \( h = \text{round}(g(w_w)/c(\mathbf{v}_i - \mathbf{d}_i - b_i \mathbf{1}_n)) \). The search process will be terminated if \( \hat{\mathbf{u}}_i^{(n_i)} = \mathbf{u}_i^{(n_i)} \), and \( \hat{\mathbf{x}}_i^{(n_i)} \) is the estimated coordinate of the \( i \)th target. Otherwise, we continue to search until all possible values of \( u_i \) are enumerated. The localization algorithm based on the brute-force search for \( u_i \), combined with the LS estimator is shown in Algorithm 1.

We remark here the observation matrix \( \mathbf{A} = [S', \hat{\mathbf{v}}, \mathbf{1}_n] \) based on (19) should be full rank to guarantee the identifiability of \( x_i \) by the LS estimator. Hence, the moving trajectory of the mobile anchor should not be linear.

**V. SIMULATION AND RESULTS**

In this section, the performance of mDRIPS is evaluated from three aspects: frequency estimation by the ESPRIT-type algorithm, ranging by the LS estimator and localization through the Algorithm 1. The median absolute error (MAE) is used as a performance metric in the simulations to mitigate the effects of the outliers due to deep fading. The amplitude of each dual-tone signal is \( a=1 \) and the carrier frequency \( f_c=434 \) MHz. We assume three active targets \( (I=3\text{ and }i=0,1,2) \), and the basic frequency of each dual-tone signal is \( f_c=1 \) MHz, \( f_t=1.1 \) MHz, \( f_c=1.2 \) MHz. The frequency difference \( g_s=50 \) kHz, which is smaller than the typical channel coherence bandwidth at the VHF band [22]. The directly sampling frequency \( f_s=3 \) MHz. Moreover, the average channel power of the flat-fading channel is assumed to be 1, thus \( \sigma_i^2 = 1 \). In the simulations, we assume that the noise term is modeled as a zero mean complex
Gaussian random process. The signal-to-noise ratio (SNR) is defined as $1/\sigma^2$, where $\sigma^2$ is the variance of the noise term. The clock skew $\nu$ varies from each Monte Carlo run within a range of [1-10 ppm, 1+10 ppm], (1 ppm=10^{-6}). Note that this bound can cause a frequency offset up to 4.35 kHz in the mDRIPS. Hence, there is a sufficient guard band to avoid frequency aliasing problem with the frequency allocation. The sampling duration $T_s$ and time slot period $T$ of the mobile anchor are 1 ms and 10 s, respectively. For each evaluation, 1000 Monte-Carlo runs are carried out.

### 5.1 Estimation accuracy of ESPRIT

In this subsection, we present the frequencies and clock skew estimation accuracy for all targets. Due to the target oscillator instability, the dual-tone signal frequencies $f_1^r, f_2^r$ are estimated by the ESPRIT-type algorithm before the ranging and localization steps. The true clock skew $\nu =1+10$ ppm, and the number of collected samples $M =1024$. We let $p=200$ and $q=M-p+1$ to balance the estimation accuracy and computation complexity. The MAE of frequency estimates versus SNR is indicated in Fig.3. They are in the range of $[10^{14}$ Hz, $10^{24}$ Hz] at high SNR. The MAE of $\hat{f}_1^r$ is slightly smaller than $\hat{f}_2^r$ because the true value of $f_1^r$ is less than $f_2^r$. Similarly, the estimation error is larger for the target with higher frequencies of the dual-tone signal. As shown in Fig. 3, the MAE of clock skew estimation is in the range of $[10^{8}$, $10^{12}]$. The accurate estimation of the tone frequencies is crucial for phase estimation and ranging.

### 5.2 Ranging accuracy

In this subsection, we investigate the ranging accuracy of mDRIPS for synchronous targets using the estimated frequencies. We assume that there is no clock offset of the target ($A=0$) and $\nu=0$. The true clock skew $\nu =1+10$ ppm. Using the estimated parameters $f_1^r, f_2^r, w_i$ is used as a benchmark. The comparison of MAEs of distance estimates versus SNR using the estimated, the accurate and the nominated $f_1^r, f_2^r, w_i$ are indicated in Fig. 4, respectively. Using the nominated $f_1^r, f_2^r, w_i$, the ranging always fails even at high SNR. On the other hand, using the estimated $f_1^r, f_2^r, w_i$, the ranging performance can achieve high accuracy at high SNR. The performance gap between the benchmark and the ones using the estimated parameters is less than 5 dB. Moreover, the ranging error with estimated parameters is larger for the target with higher frequencies of dual-tone signal. Since the estimation error of frequencies is larger, which can also be seen from Fig. 3. However, the ranging performance is similar for all targets with accurate frequencies.

### 5.3 Localization accuracy

The localization performance is evaluated in a 6 km×6 km rectangle to simplify the integer ambiguity issue. Hence, $k=0$ in (17). The three targets’ coordinates are fixed at $x_1=[1.5, 2]$ km, $x_2=[1.6, 2]$ km, $x_3=[1.7, 2]$ km and the mobile anchor original position $s_0=[3, 3]$ km. After

![Fig.3 MAE of frequencies, clock skew estimates vs. SNR](image-url)
ranging at previous position, the mobile anchor will move to the next position with a speed \(v=30 \text{ m/s}\). The anchor chooses the moving directions randomly for simplicity in this simulation. In each Monte-Carlo run, the clock offset of the \(i\)th target \(A_i\) is uniformly distributed in the range of \([0,1]\) s, and the target clock skew \(w_i\) is uniformly distributed in the range of \([-10 \text{ ppm}, 1+10 \text{ ppm}]\). The mobile anchor begins to receive signals after all targets transmitting, and the first receiving time instant \(t_0\) is set as 1.2 s. The mobile anchor takes 8 periods \((N=8)\) to finish the localization steps. For each target, after measuring 8 TOAs along with the mobile anchor’s trajectory. The TOAs will be used to locate the corresponding targets. The comparisons of MAEs of position estimates versus SNR using the nominated, estimated, and accurate \(f_i, f_f, w_i\) are indicated in Fig. 5, respectively. Without frequency estimation, the localization algorithm fails because of the huge ranging errors. However, localization based on the estimated frequencies and clock skew can approximately achieve the accuracy similarly as the ones based on accurate parameters. We can also see that the localization accuracy with estimated \(f_i, f_f, w_i\) is slightly different for different targets. The target with the largest dual-tone signal performs a little worse than other two targets because of the frequency estimation errors. Nevertheless, the localization accuracy with accurate \(f_i, f_f, w_i\) is almost the same for each target.

**VI. CONCLUSIONS**

This paper has proposed a low-cost dual-tone radio interferometric positioning system using a single mobile anchor to locate multiple targets at the same time, named mDRIPS. The mDRIPS is robust to noise, flat-fading channels and target oscillator instability. No time synchronization between different targets is required. The special design of the dual-tone signal allows the mDRIPS to accommodate flat-fading channels. Moreover, the challenges of clock offset and clock skew for localization are considered and an ESPRIT-type algorithm is designed to estimate the tone frequencies of the target. Furthermore, we investigate the integer ambiguity problem due to phase wrapping and develop a localization algorithm to deal with the ambiguity problem. The simulation results demonstrate that the localization accuracy of mDRIPS can achieve dozens of centimeters at high SNR.
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References


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